# Comparing Discrete and Continuous Markov Models for Credit Risk

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## Introduction

### Topic

The goal of this research is to use mathematical modeling to analyze and predict credit risk over time. In particular, the focus is on how corporate bond credit ratings change and how this evolution may be modeled to calculate the likelihood that each credit rating will default, as well as the estimated financial loss in a sample corporate bond portfolio. Building and analyzing this model can provide valuable insight into the relationship between default rates and the associated risk assessment of a corporate bond.

A corporate bond is a debt obligation that is used to raise capital, in which investors who buy them lend money to the company issuing the bond (U.S. Securities and Exchange Commission, 2013). Rather than receiving equity, bond owners receive interest and the principal on the bond. In the event of default, the situation in which a company fails to make its interest payment on time, bondholders are legally required to be paid before shareholders. A sample listing of an investment-grade bond is shown in Table 1. Key properties include price, coupon rate (the rate of interest paid on the bond), maturity (the time at which repayment to the bondholder is due), yield to maturity (the expected return on investment at maturity), and credit rating.

**Table 1**

*Sample Bond Listing*

| Description | Coupon (%) | Maturity | Price ($) | Yield to Maturity (%) | S&P Rating |
|---|---|---|---|---|---|
| ORACLE CORPORATION | 1.65 | 2026-03-25 | 97.91 | 4.58 | BBB |

**Note.** Coupon rates are expressed as decimals. Maturity is in years. The portfolio consists of 33 bonds total; 32 bonds are not shown. Adapted from "Corporate Ladder 1-5 Year Sample Portfolio at a Glance," by BlackRock (2025)

The general area of interest for this project is risk analysis. Like all investment assets, investing in bonds carries risk. The most notable is the default risk. Credit risk is typically evaluated as an assessment of credit exposure, probability of default, and the loss incurred in the event of default (Malhotra, 2022). Credit agencies, such as S&P Global, Moody's, and Fitch Ratings, use credit ratings to determine the borrower's ability to fulfill their debt obligations. S&P Global assigns ratings from AAA (investment grade) to CCC (near default). If a company is considered more (less) likely to pay its debts, its associated credit rating can be transitioned to a new rating. Therefore, for any given bond at any time before maturity, the credit rating can remain the same, upgrade, downgrade, or default. Credit rating agencies typically publish these outcomes in an annual transition matrix, showing the probability of transitioning from one credit level to another. These probabilities are derived from historical data. Table 2 lists the one-year average transition matrix published by S&P Global (2024).

**Table 2**

*Average Multiyear Global Corporate Transition Matrices, 1981–2024 (%)*

| From / To | AAA | AA | A | BBB | BB | B | CCC/C | D | NR |
|---|---|---|---|---|---|---|---|---|---|
| AAA | 87.28 | 8.92 | 0.51 | 0.03 | 0.10 | 0.03 | 0.05 | 0.00 | 3.08 |
| AA | 0.45 | 87.74 | 7.50 | 0.44 | 0.05 | 0.06 | 0.02 | 0.02 | 3.74 |
| A | 0.02 | 1.48 | 89.42 | 4.64 | 0.23 | 0.10 | 0.01 | 0.05 | 4.04 |
| BBB | 0.00 | 0.07 | 3.05 | 87.33 | 3.21 | 0.40 | 0.09 | 0.14 | 5.71 |
| BB | 0.01 | 0.02 | 0.10 | 4.44 | 78.89 | 6.25 | 0.50 | 0.56 | 9.23 |
| B | 0.00 | 0.02 | 0.06 | 0.14 | 4.47 | 75.18 | 4.79 | 2.93 | 12.41 |
| CCC/C | 0.00 | 0.00 | 0.07 | 0.13 | 0.40 | 13.18 | 45.07 | 26.12 | 15.03 |

**Note.** Adapted from S&P Global Market Intelligence's CreditPro and S&P Global Ratings Credit Research & Insights (2024).

Credit transition matrices and the probability of default have well-known practical uses in credit risk management. One such example is determining the credit risk value at risk (VaR). VaR is the maximum expected loss that is not likely to be exceeded at a given confidence level.

Credit VaR is defined similarly for credit loss. It is used in the regulation of capital requirements for credit risk or as an industry-wide metric for internal credit risk management (Hull, 2015). Other techniques, such as estimating expected loss (a calculation based on the probability of default and estimated recovery rate), assessing the volatility of less (unexpected loss), and evaluating portfolio diversification, are core components of assessing credit risk and are often used in calculating the credit VaR (RiskMetrics Group, 2007).

## Research Question

The goal of this project is to investigate the effectiveness of two Markov-based models in evaluating credit risk. Specifically, the research question is *How does the use of a continuous-time Markov process compare to a discrete-time Markov chain in estimating cumulative default probabilities and portfolio-level credit value at risk over a multi-year horizon?*

The availability of historical credit rating transition matrices and established credit VaR methodologies allows for the implementation of a robust and analytical model within the scope of this project. The analysis involves evaluating the non-absorbing steady state in comparison to the overall debt market, estimating cumulative probability of default for each credit rating over a 10-year time horizon, determining portfolio-level credit VaR, and examining each model's sensitivity to changes in transition probabilities. Practically, the resulting model and analysis can be used for a better understanding of real-world credit risk and its implications for various financial measures. For example, modeled probabilities of default can be used to price credit default swaps, an insurance against default by transferring credit risk from one party to another, providing a market-based measurement of credit risk (Hull, 2015).

## Information

The core data set to estimate the probabilities of default is the S&P Global (2024) average transition rate matrix shown in Table 2. A sample data set of a portfolio by the investment company BlackRock (2025), with the same structure found in Table 1, will be used for portfolio risk analysis. The associated credit VaR can be calculated using the RiskMetrics Group CreditMetrics method.

CreditMetrics was introduced in 1997 by the investment firm JP Morgan as a method to evaluate the associated credit VaR of a bond portfolio. It differs from other credit risk models, such as CreditRisk+ and Vasicek's model, by accounting for bond downgrades in addition to defaults (Hull, 2015). For a portfolio with two or more bonds, CreditMetrics uses the joint probability that each bond migrates to a different credit rating or defaults. The simplest method of calculating the joint probabilities is to assume that it is the product of the two individual likelihoods. For example, the probability of two bonds staying the same rating is:

$$\Pr(\text{Both stay the same}) = \Pr(\text{Bond 1 stays the same}) \cdot \Pr(\text{Bond 2 stays the same}) \tag{1}$$

Since there are eight possible transition states, there are $8^n$ joint transition states, where $n$ is the number of bonds in the portfolio. Therefore, CreditMetrics suggests calculating the credit VaR via simulation. Research from Han (2008) outlines a Monte Carlo simulation for assessing credit VaR using stock returns, correlating stock returns with rating migrations via a threshold matrix. The complete implementation can be found in the Model Creation section.

Sufficient data and parameters are available to support this research and perform the credit VaR calculation. Both the credit transition matrix for modeling the probability of default and the average recovery rate, which determines value at default, are publicly available from S&P Global. The process for calculating the forward interest rate and future prices of a bond is outlined by Hull (2015). The credit transition matrix from S&P Global acts as the base case for one-year rating transitions, and subsequent transition probabilities are modeled with this case.

**Assessment**

The data and information gathered from this research provide a solid foundation for building, solving, and analyzing a mathematical model in the given time frame. When modeling something like credit risk, simplifying assumptions are necessary to reduce complexity while keeping the core foundation of the problem. By using the S&P Global transition matrix, we assume it accurately represents the total corporate bond market and does not consider various economic, environmental, social, and governmental factors that may influence a bond's future credit rating (Chen et al., 2025). To model credit VaR, the CreditMetrics Monte Carlo simulation assumes normally distributed stock returns and a constant recovery rate in case of default. Due to the difficulty in obtaining large volume time series data for corporate bond transitions, it is reasonable to use the available S&P Global transition matrix as the base case.

For this project, a central simplifying assumption is that credit rating transitions follow a time-homogeneous Markov process. This means that the probability of a bond transitioning from one rating to another depends only on the current rating, and the probability of moving between two states depends only on the length of the time interval (Kiefer and Larson, 2004). In a discrete time-homogeneous Markov chain, such as the base case 1-year S&P Global transition matrix, this means that the transition probabilities remain the same for the entire year. In reality, research by Kavvathas (2000) indicates that credit rating transitions exhibit non-Markovian properties. For example, there exists a property called rating momentum, where a company's credit rating being upgraded (downgraded) is more likely to be further upgraded (downgraded), invalidating the time homogeneous Markov property. While these assumptions may not account for the complete activity of the real corporate bond market, they provide a structured framework for estimating credit risk using established metrics and methodologies.

## Model

### Selection

For this research, a Markov chain model, comparing a discrete-time Markov chain with a continuous Markov process approximated by a generator matrix, $Q$, has been chosen to model credit rating transitions and assess credit risk in a corporate bond portfolio over various time horizons. This comparative approach enables the evaluation of credit migration probabilities and default risk under both a discrete and continuous setting, providing a basis for estimating credit VaR and expected loss of a corporate bond portfolio. This model accounts for probabilistic transitions between states over time and can be considered a stochastic model satisfying the Markov property, where the future state of a credit rating only depends on the current state.

While there are multiple methods for modeling probabilities of default and credit rating transitions, such as estimating default probabilities using credit spreads (Hull, 2015), a Markov model offers a simplified, tractable approach using historically published data, without requiring large volumes of detailed corporate data. Other industry supported models, such as Merton's approach, estimate the probability of default requiring firm specific information, such as asset values, volatilities, and debt structures that may not be readily available for an entire bond portfolio.

Researchers Kiefer and Larson (2004) propose a likelihood ratio test for determining whether a credit rating transition matrix supports a time-homogeneous Markov model. Specifically, Kiefer and Larson suggest that corporate bonds can be modeled as a Markov chain for up to three years. However, they do not rule out using a Markov model for periods larger than three years. Applying this test to the corporate transition matrices published by S&P Global's 2024 *Annual Global Corporate Default and Rating Transition Study* gives similar three-year results as shown in Table 3.

**Table 3**

*Likelihood Ratio Test for Markov Property of Credit Rating Transitions*

| Transition Period (Years) | LR-Value | Degrees of Freedom | $\Pr(\chi^2 > \text{LR})$ |
|---|---|---|---|
| 1–2 | 30.287 | 56 | 0.998 |
| 1–3 | 72.669 | 112 | 0.0664 |
| 1–5 | 246.661 | 168 | 0.0000 |

**Note**.The null hypothesis suggests the Markov model fits well. Data are from S&P Global. Year 4 transitions are not included in the data set. Test adapted from Kiefer and Larson (2004)

## Creation

To address the research question: *How does the use of a continuous-time Markov process compare to a discrete-time Markov chain in estimating cumulative default probabilities and portfolio-level credit value at risk over a multi-year horizon?* we construct and analyze two Markov-based models to simulate credit rating transitions and calculate the resulting credit VaR. The model assumes that credit rating transitions follow a time-homogeneous Markov process with default as an absorbing state. Both models will be used to examine the evolution of transition probabilities, cumulative default rates, and the credit VaR for a sample bond portfolio consisting of 33 corporate bonds of mixed ratings.

For both models to be a time-homogeneous Markov process, they must satisfy the Markov property. Let $\{X(t)\}_{t\geq 0}$ be a stochastic process that forms a sequence of random variables $X_0, X_1, \ldots X_t$ representing the credit rating state of a corporate bond at time $t$ with outcomes $x_0, x_1, \ldots, x_t$ on the state space $S = \{AAA, AA, BBB, BB, B, CCC, D\}$. In the discrete-time Markov chain model, we assume that credit rating transitions occur only at fixed annual intervals $t = 0, 1, 2\ldots$ The resulting Markov chain satisfying the Markov process is

$$P(X_{n+1} \mid X_0 = x_o, X_1 = x_1, \ldots X_n = x_n) = P(X_{n+1} = j \mid X_n = i) = P_{ij} \tag{2}$$

for all stages $n$ and all states $x_0, x_1, \ldots x_{n+1} \in S$. Given the time-homogeneous assumption, future rating transition probabilities over $n$ years can be calculated via matrix multiplication as

$$P\left(X_{n+k} = j \mid X_k = i\right) = (P^n)_{ij} \tag{3}$$

The empirical one-year credit rating transition matrix $P$, shown in Table 4, serves as the base case for the discrete Markov chain.

The continuous Markov process model satisfies the Markov property from (2) and defines the credit rating transition probabilities as

$$P(X(t) = j \mid X(0) = i) = (e^{Qt})_{ij} \tag{4}$$

where $Q$ is the generator matrix, satisfying

$$q_{ij} \geq 0 \text{ for all } i \neq j, \quad \text{and} \quad q_{ii} = -\sum_{j\neq i} q_{ij} \tag{5}$$

The resulting generator $Q$ calculated using the Jarrow et al. (1997) logarithmic approximation method from $P$ is shown in Table 5, and the transition matrix for one year calculated using this generator is shown in Table 6.

**Table 4**

*Adjusted Credit Rating Transition Matrix from AAA to D (S&P Global)*

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 0.9005 | 0.0920 | 0.0053 | 0.0003 | 0.0010 | 0.0003 | 0.0005 | 0.0000 |
| AA | 0.0047 | 0.9115 | 0.0779 | 0.0046 | 0.0005 | 0.0006 | 0.0002 | 0.0002 |
| A | 0.0002 | 0.0154 | 0.9318 | 0.0484 | 0.0024 | 0.0010 | 0.0001 | 0.0005 |
| BBB | 0.0000 | 0.0007 | 0.0323 | 0.9262 | 0.0340 | 0.0042 | 0.0010 | 0.0015 |
| BB | 0.0001 | 0.0002 | 0.0011 | 0.0489 | 0.8691 | 0.0689 | 0.0055 | 0.0062 |
| B | 0.0000 | 0.0002 | 0.0007 | 0.0016 | 0.0510 | 0.8583 | 0.0547 | 0.0335 |
| CCC | 0.0000 | 0.0000 | 0.0008 | 0.0015 | 0.0047 | 0.1551 | 0.5304 | 0.3074 |
| D | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |

**Note.** Data represent one-year credit rating transition probabilities from AAA to D based on S&P Global ratings. NR ratings filtered using (7)

**Table 5**

*Approximated Generator Matrix Q for Credit Rating States*

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | -0.1048 | 0.0969 | 0.0055 | 0.0003 | 0.0011 | 0.0003 | 0.0005 | 0 |
| AA | 0.0049 | -0.0927 | 0.0816 | 0.0048 | 0.0005 | 0.0007 | 0.0002 | 0.0002 |
| A | 0.0002 | 0.0160 | -0.0706 | 0.0501 | 0.0025 | 0.0011 | 0.0001 | 0.0005 |
| BBB | 0 | 0.0008 | 0.0336 | -0.0767 | 0.0354 | 0.0044 | 0.0010 | 0.0015 |
| BB | 0.0001 | 0.0002 | 0.0012 | 0.0524 | -0.1403 | 0.0738 | 0.0059 | 0.0066 |
| B | 0 | 0.0002 | 0.0007 | 0.0017 | 0.0550 | -0.1528 | 0.0590 | 0.0361 |
| CCC | 0 | 0 | 0.0011 | 0.0021 | 0.0064 | 0.2095 | -0.6341 | 0.4151 |
| D | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Note.** Generator matrix $Q$ corresponding to credit rating states. Derived from S&P Global transition matrix $P$ using logarithmic approximation (10).

To evaluate portfolio-level credit VaR for each model, we perform a Monte Carlo simulation using the CreditMetrics asset correlation model in accordance with Han (2008). In this

**Table 6**

*1-Year Transition Matrix from Continuous-Time Markov Model*

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 0.9008 | 0.0879 | 0.0087 | 0.0007 | 0.0010 | 0.0004 | 0.0004 | 0.0001 |
| AA | 0.0044 | 0.9123 | 0.0753 | 0.0063 | 0.0007 | 0.0007 | 0.0002 | 0.0003 |
| A | 0.0002 | 0.0148 | 0.9332 | 0.0467 | 0.0031 | 0.0012 | 0.0001 | 0.0006 |
| BBB | 0 | 0.0010 | 0.0313 | 0.9278 | 0.0319 | 0.0052 | 0.0009 | 0.0019 |
| BB | 0.0001 | 0.0003 | 0.0019 | 0.0472 | 0.8717 | 0.0645 | 0.0057 | 0.0086 |
| B | 0 | 0.0002 | 0.0008 | 0.0029 | 0.0478 | 0.8646 | 0.0404 | 0.0432 |
| CCC | 0 | 0 | 0.0009 | 0.0018 | 0.0087 | 0.1433 | 0.5343 | 0.3110 |
| D | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0000 |

**Note.** This transition matrix represents the estimated 1-year probabilities of moving between credit rating categories, based on the continuous-time Markov chain model using generator $Q$ approximated with $P$.

model, the credit rating transition matrices calculated by each Markov model are linked with the stock return data found in Table 7. First, we can calculate future prices of each bond by using the coupon and current price to determine spot rates and estimate future interest rates. Table 8 shows the forward zero rates for each credit rating (excluding default) from 1 to 5 years. Using the forward rate, we determine the forward value for each bond in the portfolio, assuming a recovery of 51% in the event of default (RiskMetrics Group, 2007). The forward values of each bond in the portfolio are presented in Table 9. By assuming the data in Table 7 is normally distributed, we determine Z asset return thresholds via the inverse normal cumulative distribution function. A sample Z asset return threshold matrix for the discrete model is shown in Table 10.

**Table 7**

*Annual Stock Returns for Selected Portfolio (2015–2024)*

| Year | BNS (%) | ORCL (%) | FI (%) |
|---|---|---|---|
| 2015 | -23.38 | -16.44 | 29.51 |
| 2016 | 45.82 | 8.77 | 18.75 |
| 2017 | 18.98 | 24.13 | 22.16 |
| 2018 | -19.92 | -1.62 | 12.76 |
| 2019 | 16.51 | 19.16 | 60.13 |
| 2020 | 1.28 | 22.02 | -1.44 |
| 2021 | 41.63 | 38.91 | -7.26 |
| 2022 | -28.58 | -5.39 | -6.09 |
| 2023 | 7.24 | 27.84 | 31.25 |
| 2024 | 18.54 | 62.10 | 54.36 |

**Note.** Annual total returns for three companies in the portfolio. The portfolio is adapted from BlackRock (2025) and consists of 33 bonds total; 30 bonds are not shown. Data is collected from Yahoo Finance

**Table 8**

*Forward Rates by Rating Category (Years 1–5)*

| Rating | $F_{12}$ | $F_{13}$ | $F_{14}$ | $F_{15}$ |
|---|---|---|---|---|
| AAA | 0.0435 | 0.0445 | 0.0414 | 0.0402 |
| AA | 0.0559 | 0.0544 | 0.0549 | 0.0560 |
| A | 0.0621 | 0.0642 | 0.0642 | 0.0634 |
| BBB | 0.0690 | 0.0715 | 0.0720 | 0.0736 |
| BB | 0.0765 | 0.0796 | 0.0832 | 0.0831 |
| B | 0.0843 | 0.0882 | 0.0926 | 0.0933 |
| CCC | 0.1000 | 0.1055 | 0.1091 | 0.1051 |

**Note.** Values represent implied annualized forward interest rates for years 2 through 5, computed from (14)

**Table 9**

*Forward Bond Prices for One Year (Sample of 3 Bonds)*

| Rating | BNS | ORCL | FI |
|---|---|---|---|
| AAA | 105.1354 | 99.0646 | 102.1000 |
| AA | 103.9577 | 97.9217 | 100.9397 |
| A | 103.3710 | 97.3524 | 100.3617 |
| BBB | 102.7419 | 96.7419 | 99.7419 |
| BB | 102.0587 | 96.0790 | 99.0689 |
| B | 101.3528 | 95.3939 | 98.3733 |
| CCC | 99.9773 | 94.0591 | 97.0182 |
| D | 51.0000 | 51.0000 | 51.0000 |

**Note.** Table shows forward bond prices for three representative bonds from a 33-instrument portfolio calculated using (15). All prices are in dollars.

**Table 10**

*Discrete Model Year One Z-Threshold Matrix Based on Adjusted S&P Transition Matrix*

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | - | - | - | - | - | - | - | - |
| AA | -1.285 | 2.599 | 3.530 | - | 3.695 | - | - | - |
| A | -2.436 | -1.380 | 2.154 | 3.178 | 3.406 | 3.505 | - | - |
| BBB | -2.853 | -2.517 | -1.621 | 1.837 | 2.982 | 3.117 | 3.147 | - |
| BB | -2.902 | -3.000 | -2.638 | -1.742 | 1.641 | 2.806 | 2.826 | - |
| B | -3.147 | -3.145 | -2.916 | -2.474 | -1.401 | 1.611 | 2.454 | - |
| CCC | -3.283 | -3.530 | -3.183 | -2.815 | -2.268 | -1.352 | 0.986 | - |
| D | - | - | -3.227 | -2.971 | -2.502 | -1.832 | -0.503 | - |

**Note.** The Z-thresholds for transition testing are based on the normal distribution of stock returns and probabilities from the adjusted S&P credit rating matrix in Table 4.

Using the normally distributed stock returns, we sample 10,000 portfolio returns via a Monte Carlo simulation with mean and covariance as inputs, and standardize to get a normal distribution of return $X = (X-\mu)/\sigma$. Each simulation is compared against the asset return Z threshold matrix to simulate rating transitions and is revalued accordingly. The data is sorted in ascending order, and the credit VaR is calculated as

$$\text{C-Var} = V_{P,5\%} - \overline{V_p} \tag{6}$$

where $V_{P,5\%}$ is the portfolio value at the top 5 percent, and $\overline{V_p}$ is the sample mean.

**Process**

The construction of both Markov models used to estimate transition probabilities between credit ratings was based on both financial research and industry practices. Assuming time homogeneity, the S&P Global credit rating transition matrix can be used as a discrete model to calculate future transition probabilities. The adjusted transition matrix in Table 4, used as the base case for the discrete model, had the not-rated categories adjusted from Table 2, using the method from Jarrow et al. (1997).

$$\hat{P}_{ij} = \frac{P_{ij}}{1-P_{i,\text{NR}}} \quad \text{for } j \neq \text{NR} \tag{7}$$

where $\hat{P}_{ij}$ represents the adjusted value, removing the not-rated (NR) category. As stated previously, research by Kiefer and Larson (2004) suggests that a Markov model is a strong fitting model for up to three years, and can be used with caution for subsequent years.

In practice, credit risk analysts typically use a continuous Markov model for short-term credit risk, such as valuing the price of a credit default swap (Israel et al., 2001). Published transition matrices, such as the one in Table 4, are estimated on a yearly interval, and due to a small pool of rating transitions, they cannot reliably be constructed for periods of less than a year. To convert a transition matrix to a continuous model, the evolution of state probabilities follows the Kolmogorov forward equation (Kavvathas, 2000)

$$\frac{dp(t)}{dt} = p(t)Q \tag{8}$$

where $p(t)$ is a row vector and $p_i(t)$ represents the probability that a bond is in state $i$ at time $t$, and $Q$ is the generator matrix. This forms a linear system of differential equations with a nonlinear solution

$$p(t) = e^{Qt} \text{ approximated by } p(t) = \sum_{k=0}^{\infty} \frac{(tQ)^k}{k!} \tag{9}$$

Therefore, $p(t)$ is a time-dependent probability function over the state space of credit ratings, satisfying the Markov property.

Not every transition matrix $P$ corresponds to a valid generator $Q$. The scenario in which $Q$ exists means that $P$ is embeddable. In the case that there exists $i$ and $j$ such that $j$ is accessible from $i$, but $p_{ij} = 0$, then $P$ is not embeddable (Israel et al., 2001). Looking at the matrix from Table 2, we see that $p_{1,2} = 0.0920 > 0$, $p_{2,8} = 0.0002 > 0$, but $p_{1,8} = 0$. Therefore, the matrix $P$ is not embeddable, and the generator must be approximated as a best fit. The simplest approximation that satisfies all the conditions of embeddability is the formula:

$$q_{ii} = \log(p_{ii}) \quad q_{ij} = p_{ij} \cdot \log(p_{ii})/(p_{ii} - 1) \tag{10}$$

derived by Jarrow et al. (1997), and was used to calculate the generator matrix shown in Table 5.

The Monte Carlo simulation used to evaluate these Markov models on a bond portfolio is an industry standard following the CreditMetrics methodology. The core idea is to correlate a company's annual stock return with credit rating transition probabilities using a bivariate normal distribution. Assuming that $R$ is a stock return from a normally distributed set of stock returns, the following relationship is established

$$\Pr(\text{default}) = \Pr(R < Z_{def}) = \Phi(Z_{def}/\sigma) \tag{11}$$

where $\Phi$ is the cumulative normal function. The corresponding default threshold value $Z_{def}$ can be determined by

$$Z_{def} = \Phi^{-1}(\Pr(\text{default})) \tag{12}$$

Spot rates, future interest rates, and forward values of the bonds in the portfolio are standard financial calculations and can be found in Han (2008), and Hull (2015). Spot rates, the implied interest rate from today until year $j$, are calculated as

$$s_{ij} = \left(\frac{FV}{P_{ij}}\right)^{1/j} - 1 \tag{13}$$

where $FV$ is the face value of the bond, and $P_{ij}$ is the price of a bond with rating $i$ and maturity $j$. Using the spot rates, we calculate the forward rate, the implied interest rate between future periods, using the formula

$$f_{1,j} = \left( \frac{(1+s_j)^j}{1+s_1} \right)^{1/(j-1)} \tag{14}$$

where $s_j$ is the spot rate at year $j$. Finally, each bond in the portfolio can be revalued as

$$V_{X,A} = \sum_{i=1}^{n} \frac{C}{(1+F_i)^{i-1}} + \frac{C}{(1+F_n)^n} \tag{15}$$

where $V_{X,A}$ is the market value of the bond $X$ at rating $A$. $C$ is the coupon of the bond, and $F$ is the forward rate of rating $A$ bond from year 1 to $n$. Using these values, we reevaluate the portfolio using the future values in each simulation and calculate the estimated credit VaR.

**Tools**

The primary tool used for creating and analyzing the model was MATLAB, chosen for its functionality with linear algebra, matrix operations, and random sampling over probability distributions. The transition matrix from Table 2 was written into MATLAB, and then adjusted via (7) to produce the matrix $P$ shown in Table 4. The generator was created using the log approximation from (10) on $P$. Future probabilities were modeled using MATLAB's matrix operations. For example, future probabilities for the continuous model were calculated using *expm(Q*t)* to represent $e^{Qt}$ from (4). Steady states and cumulative default rates were also calculated using MATLAB's linear algebra tools, such as *linsolve*, used to solve the linear system for steady states.

The CredtiMetrics Monte Carlo simulation was also conducted using MATLAB. The asset return Z threshold matrix can easily be computed in MATLAB using the *cumsum* and *norminv* functions for each row. Random sampling of the multivariate distribution by Han (2008) is also trivial using MATLAB's *mvrnd* function, which allows for the mean and covariance vectors to be included as inputs. The associated Credit VaR is then calculated using the *sort* function and (6).

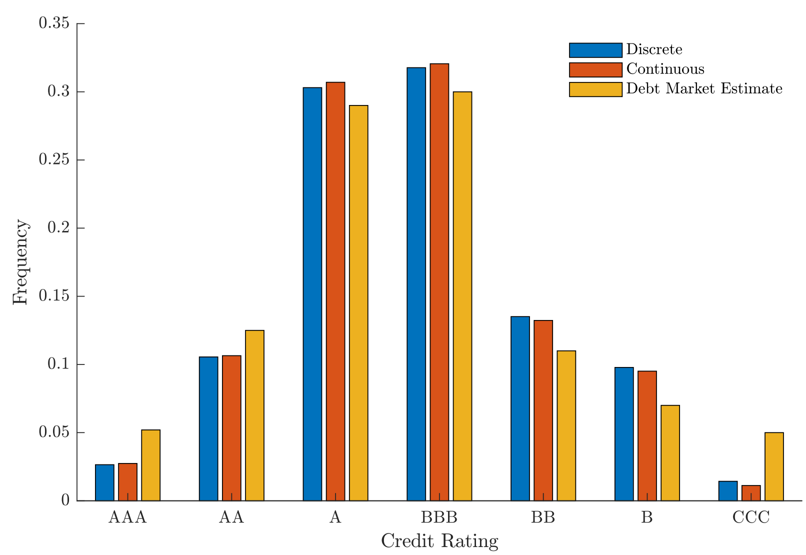
Alongside calculation, MATLAB was also used to formulate tables and figures. Results from modeling cumulative default rates, $L^1$ norm, and the steady state were transformed into their appropriate figures, providing clear visual comparisons between the two models. Data stored in vectors can easily be converted into tables, alongside any accompanying statistics, such as percent change and MAPE.

## Analysis

To assess long-term behavior, the steady-state distributions were calculated for both models. Because default is an absorbing state, the steady-state specifically focuses on the non-absorbing survival distribution. As shown in Figure 1, compared to the discrete model, the continuous model estimates slightly more investment grade (AAA-BBB) and fewer speculative (BB-CCC) bonds. Both models closely resemble the current debt market estimate from S&P Global (2024); however, they severely underestimate both AAA and CCC bonds. For improved credit risk estimates under current market conditions, both models could be adjusted for higher steady states of AAA and CCC ratings.

**Figure 1.**

*Steady State Probabilities*




Model similarity was evaluated using Kullback-Leibler (KL) divergence and the $L^1$ norm. KL divergence measures the distance between the two models in terms of how they represent rating transitions, and the $L^1$ norm corresponds to the total absolute difference between the two models. The results from the KL divergence in Table 11 show that the continuous model's generator $Q$ produced transition probabilities with a similar distribution to the discrete matrix $P$.

**Table 11**

*KL Divergence Between Discrete and Continuous Methods by Rating Category*

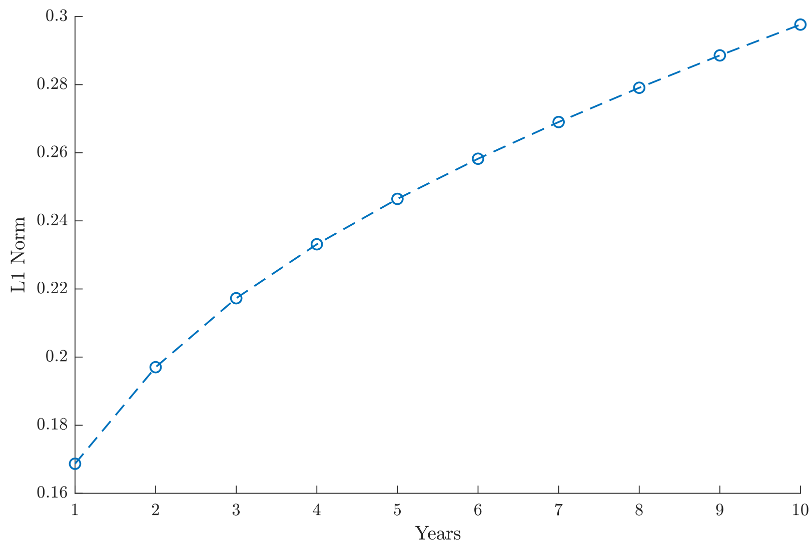
| Rating | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 | Year 8 | Year 9 | Year 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| AAA | 0.00118 | 0.00136 | 0.00139 | 0.00136 | 0.00132 | 0.00126 | 0.00121 | 0.00116 | 0.00112 | 0.00108 |
| AA | 0.00035 | 0.00048 | 0.00052 | 0.00053 | 0.00052 | 0.00051 | 0.00050 | 0.00049 | 0.00049 | 0.00049 |
| A | 0.00015 | 0.00021 | 0.00025 | 0.00028 | 0.00030 | 0.00032 | 0.00034 | 0.00037 | 0.00039 | 0.00042 |
| BBB | 0.00026 | 0.00038 | 0.00046 | 0.00052 | 0.00058 | 0.00064 | 0.00069 | 0.00075 | 0.00081 | 0.00086 |
| BB | 0.00077 | 0.00122 | 0.00157 | 0.00184 | 0.00203 | 0.00215 | 0.00222 | 0.00225 | 0.00225 | 0.00224 |
| B | 0.00395 | 0.00467 | 0.00453 | 0.00416 | 0.00376 | 0.00340 | 0.00308 | 0.00281 | 0.00259 | 0.00240 |
| CCC | 0.00166 | 0.00130 | 0.00102 | 0.00091 | 0.00096 | 0.00108 | 0.00121 | 0.00130 | 0.00133 | 0.00131 |

**Note.** KL divergence between the discrete and continuous models over 10 years. Higher values indicate greater divergence.

The $L^1$ norm in Figure 2 shows that the absolute difference between the models increases gradually but remains small over a 10-year horizon, indicating a close overall fit for early years. While the $L^1$ norm increases over time, reflecting increased absolute differences in transition probabilities, the KL divergence values remain consistently small. This suggests that while the continuous model may over- or underestimate transition probabilities, the rating level probability distributions closely fit the discrete model, making it a reliable approximation.

**Figure 2.**

*$L^1$ Norm Between Discrete and Continuous Transition Models by Year*




Cumulative default probabilities were calculated over a 10-year horizon for each credit rating to compare long-term credit risk across rating categories. Shown in Figure 3, the continuous model estimates higher cumulative default probabilities across all rating categories. From Table 12, the continuous model consistently overestimates the discrete model. As time increases, and bonds approach the absorbing default state, the overestimation between the two models gradually begins to decrease.

**Figure 3.**

*Cumulative Probability of Default*

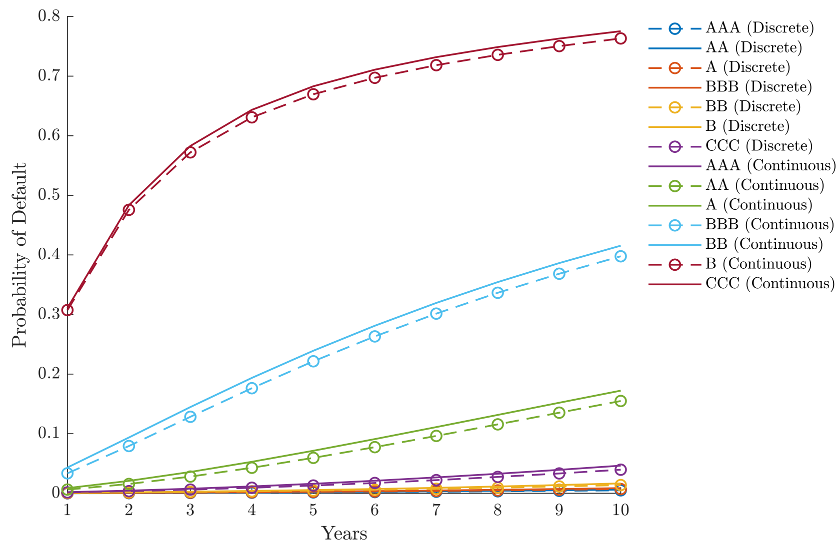



**Table 12**

*Percentage Overestimation of Cumulative Default Probabilities in Continuous Model by Rating*

| Rating | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 | Year 8 | Year 9 | Year 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| AAA | – | 102.0% | 54.55% | 39.49% | 32.45% | 28.55% | 26.17% | 24.65% | 23.63% | 22.94% |
| AA | 37.92% | 30.66% | 27.16% | 25.28% | 24.22% | 23.59% | 23.21% | 22.94% | 22.74% | 22.55% |
| A | 15.23% | 17.55% | 19.10% | 20.10% | 20.71% | 21.01% | 21.08% | 20.98% | 20.74% | 20.41% |
| BBB | 27.13% | 26.04% | 25.08% | 24.03% | 22.89% | 21.71% | 20.53% | 19.39% | 18.30% | 17.29% |
| BB | 39.89% | 33.34% | 27.82% | 23.48% | 20.12% | 17.49% | 15.41% | 13.73% | 12.36% | 11.23% |
| B | 29.24% | 18.04% | 12.76% | 9.82% | 8.00% | 6.80% | 5.95% | 5.33% | 4.85% | 4.48% |
| CCC | 1.16% | 1.63% | 1.89% | 1.99% | 2.00% | 1.95% | 1.87% | 1.78% | 1.69% | 1.59% |

**Note.** Values represent percentage overestimation of cumulative default probabilities by the continuous method compared to the discrete method. An outlier of 11,553% was excluded in Year 1 AAA bonds, due to the extremely small discrete probability of default $\left(1 \times 10^{-7}\right)$.

Figure 4 shows the mean cumulative probability of default over 10 years. As observed, the differences between the models are relatively small for the first three years, but begin to diverge in later years. Like the credit-level cumulative default probabilities, the divergence starts to decrease as bonds approach the absorbing default state. This can be seen in Table 13, where the percentage difference begins to decline from 2.512% in year 8 to 2.508% in year 10. While some significant differences exist in the rating-level cumulative default probabilities, due to the approximation of the generator $Q$, the similarity in mean cumulative default rates indicates that both models provide similar estimates of credit risk, with the continuous model being slightly more conservative overall.

**Figure 4.**

*Mean Cumulative Probability of Default*

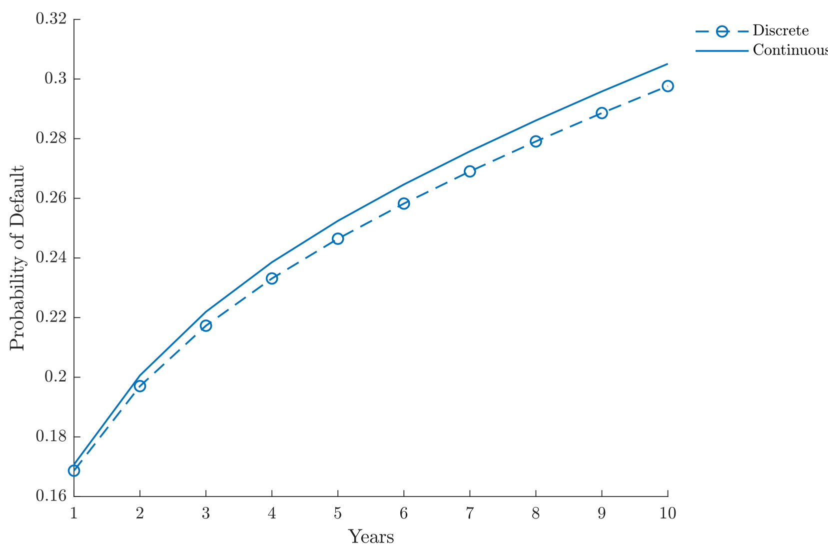


**Table 13**

*Comparison of Mean Cumulative Default Rates Between Discrete and Continuous Models*

| Year | Discrete Model | Continuous Model | Difference | % Difference |
|---|---|---|---|---|
| 1 | 0.1687 | 0.1707 | 0.0021 | 1.224% |
| 2 | 0.1970 | 0.2006 | 0.0036 | 1.826% |
| 3 | 0.2173 | 0.2220 | 0.0047 | 2.155% |
| 4 | 0.2331 | 0.2386 | 0.0054 | 2.337% |
| 5 | 0.2465 | 0.2525 | 0.0060 | 2.435% |
| 6 | 0.2582 | 0.2647 | 0.0064 | 2.484% |
| 7 | 0.2690 | 0.2758 | 0.0067 | 2.506% |
| 8 | 0.2791 | 0.2861 | 0.0070 | 2.513% |
| 9 | 0.2886 | 0.2958 | 0.0073 | 2.512% |
| 10 | 0.2976 | 0.3051 | 0.0075 | 2.508% |

**Note.** Mean cumulative default rates are averaged across all rating categories. The percent difference is calculated relative to the discrete model.

The CreditMetrics Monte Carlo simulation was then conducted to estimate the portfolio-level credit risk. Using 10,000 simulated samples from a multivariate distribution of historical asset returns, each bond was compared against an asset return Z matrix based on the normal inverse function and the modeled transition ratings. The credit VaR was calculated at the 95% confidence level, representing the maximum expected amount lost with a 5% chance that actual losses will exceed this amount. Table 14 shows the results of the simulation over 33 bonds. A year 1 simulated value of -33.47 means that within one year, the portfolio is not expected to lose more than $33.47 with 95% confidence. Across all time horizons, the continuous model produced credit VaR estimates 7.5% higher on average, aligning with the conservative estimate seen in the mean cumulative default probabilities.

**Table 14**

*95% Credit VaR: Discrete vs Continuous Model Comparison*

| Year | Discrete VaR ($) | Continuous VaR ($) | Abs. Difference | % Change |
|---|---|---|---|---|
| 1 | -33.47 | -33.38 | 0.09 | -0.27% |
| 2 | -35.40 | -36.39 | 0.98 | 2.77% |
| 3 | -57.15 | -60.82 | 3.67 | 6.42% |
| 4 | -87.47 | -95.66 | 8.19 | 9.36% |
| 5 | -116.59 | -127.03 | 10.44 | 8.96% |
| 6 | -147.35 | -160.41 | 13.06 | 8.86% |
| 7 | -177.92 | -192.06 | 14.14 | 7.95% |
| 8 | -210.12 | -225.28 | 15.16 | 7.21% |
| 9 | -244.98 | -259.30 | 14.32 | 5.85% |
| 10 | -315.96 | -372.23 | 56.27 | 17.81% |
| **MAPE: 7.55%** | | | | |

**Note.** Calculated using Monte Carlo CreditMetrics simulation via modeled credit rating transition matrices. Simulated with 10,000 simulations for 33 bonds.

Sensitivity analysis was conducted on both models to examine the effects of economic conditions on credit risk analysis. The 2008 global corporate bonds transition matrix from S&P Global was chosen as the recession matrix, and the 2021 transition matrix from S&P Global was chosen as the expansion matrix. The $L^1$ norm for each economic condition is shown in Figure 5, with the recession scenario having consistently larger $L^1$ norms, while the expansion scenario has the lowest divergence across all 10 years. The average $L^1$ norm values are 0.1803 (baseline), 0.0967 (expansion), and 0.2640 (recession). This corresponds to a ratio of approximately 2.7069 between recession and expansion, highlighting how much more sensitive the continuous model becomes under weakening economic conditions.

**Figure 5.**

*$L^1$ Norm by Economic Condition*

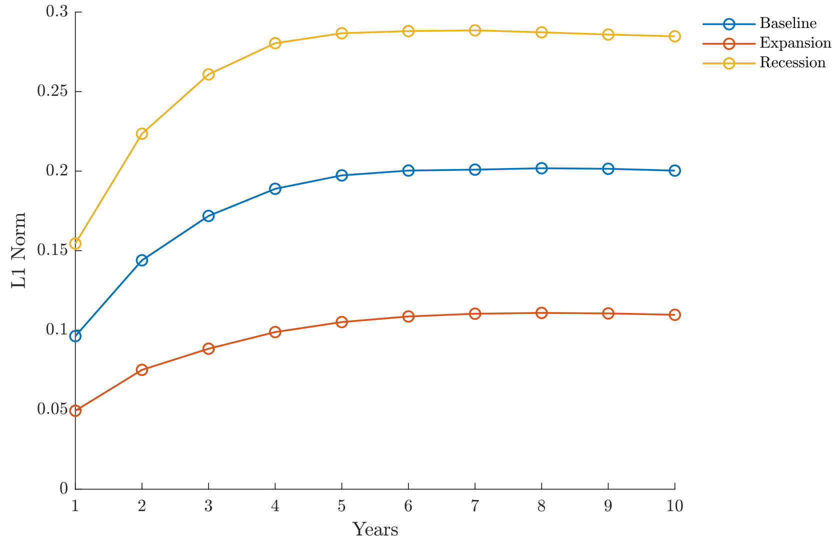



Table 15 summarizes the mean cumulative default rates for both models in a 10-year horizon under baseline, recession, and expansion conditions. For the discrete model, approximately 25% of all corporate bonds are expected to default within 10 years under the baseline. In a recession, this increases by about 16.1% to 28.50%, and decreases by 30.8% to 16.99% during economic expansion. The continuous model follows a similar estimate with greater sensitivities, with expected defaults increasing by approximately 17.7% in a recession and about a 31.9% decrease during an expansion. Both the discrete and continuous models follow the expected credit risk dynamics with each economic scenario, with increased default probability in recession and greatly reduced risk during expansion, with the continuous model showing higher sensitivity to the state of the economy.

**Table 15**

*Mean Cumulative Default Probabilities by Economic Condition and Model*

| Model | Baseline | Recession | Expansion |
|---|---|---|---|
| Discrete | 0.2455 | 0.2850 | 0.1699 |
| Continuous | 0.2512 | 0.2957 | 0.1711 |

**Note.** Mean cumulative default rates averaged over the 10-year horizon for discrete and continuous credit risk models under different economic condition.

As shown in Table 16, the continuous model generally predicts higher average cumulative default rates regardless of economic conditions. In the baseline, the continuous model predicts defaults larger than the discrete model by an average of 2.25%. This difference increases by 46% under expansion conditions, and decreases by 14% to 1.94% in a recession. This suggests that cumulative default rates of both models are more closely aligned under a recession, likely due to higher initial default rates, causing closer convergence to the absorbing state.

**Table 16**

*Average Percent Difference Between Continuous and Discrete Models and Sensitivity Ratios Relative to Baseline*

| Economic Condition | Average Percent Difference (%) | Sensitivity Ratio |
|---|---|---|
| Baseline | 2.25 | 1.00 |
| Expansion | 3.27 | 1.46 |
| Recession | 1.94 | 0.86 |

**Note.** Average percent difference is calculated as mean cumulative default rates between discrete and continuous models relative to the discrete model over a 10-year horizon. Sensitivity ratios compare each economy condition's percent difference relative to the baseline.

Table 17 shows the credit VaR sensitivity to the discrete model by economic state. As expected, the increased default probabilities from the recession matrix yield much larger estimates of loss with a mean absolute percentage error (MAPE) of 87.95%. In contrast, under expansion conditions, the expansion scenario yields estimates that initially align with the baseline but progressively diverge in later years as more rating transitions are expected to occur in the baseline model.

**Table 17**

*Discrete Model: Credit VaR Sensitivity by Economic Condition*

| Year | Baseline ($) | Expansion ($) | Recession ($) | % Change (Expansion) | % Change (Recession) |
|---|---|---|---|---|---|
| 1 | -33.47 | -32.91 | -31.36 | 1.69% | 6.30% |
| 2 | -35.40 | -35.28 | -92.96 | 0.33% | 162.59% |
| 3 | -57.15 | -35.75 | -111.19 | -37.47% | 94.54% |
| 4 | -87.47 | -49.38 | -142.59 | -43.56% | 63.02% |
| 5 | -116.59 | -62.21 | -230.04 | -46.65% | 97.32% |
| 6 | -147.35 | -79.42 | -289.62 | -46.10% | 96.50% |
| 7 | -177.92 | -89.94 | -299.66 | -49.44% | 68.39% |
| 8 | -210.12 | -108.50 | -347.60 | -48.37% | 65.41% |
| 9 | -244.98 | -114.33 | -365.93 | -53.32% | 49.36% |
| 10 | -315.96 | -126.21 | -871.87 | -60.06% | 175.99% |

**Note.** Percent change is calculated relative to the baseline.

The continuous model in Table 18 and corresponding MAPE values in Table 19 follow a similar pattern to the discrete model. The average absolute percent error under expansion conditions is 47.21% compared to 89.07% in a recession. In comparison with the discrete model, the continuous model displays an 8.5% higher error in expansion and a 1.12% increase in recession. This further suggests that regardless of economic condition, the continuous model will still provide a more conservative estimate of credit risk.

While the differences between the two models followed similar patterns as the base case, this analysis shows that cumulative probabilities of default and estimated credit VaR are extremely sensitive to economic conditions. This suggests that estimates within one year should be modeled with the state of the economy in mind. However, given that the baseline transition matrix $P$ represents an average of historical transition behavior across various economic cycles, it can still reflect a balanced midpoint for modeling credit risk without the need to account for more complex behavior.

**Table 18**

*Continuous Model: Credit VaR Sensitivity by Economic Condition*

| Year | Baseline ($) | Expansion ($) | Recession ($) | % Change Expansion | % Change Recession |
|---|---|---|---|---|---|
| 1 | -33.38 | -32.85 | -30.26 | 1.60% | 9.36% |
| 2 | -36.39 | -34.73 | -104.52 | 4.56% | 187.18% |
| 3 | -60.82 | -35.02 | -129.44 | -42.40% | 112.80% |
| 4 | -95.66 | -49.19 | -155.00 | -48.58% | 62.01% |
| 5 | -127.03 | -63.00 | -259.21 | -50.39% | 104.04% |
| 6 | -160.41 | -81.61 | -308.77 | -49.13% | 92.51% |
| 7 | -192.06 | -89.70 | -307.30 | -53.29% | 59.98% |
| 8 | -225.28 | -110.83 | -401.61 | -50.80% | 78.26% |
| 9 | -259.30 | -116.09 | -378.05 | -55.23% | 45.80% |
| 10 | -372.23 | -126.10 | -887.98 | -66.12% | 138.52% |

**Note.** Percent change is calculated relative to the baseline.

**Table 19**

*MAPE of Expansion and Recession Scenarios Compared to Baseline by Model*

| Model | Expansion vs Baseline MAPE (%) | Recession vs Baseline MAPE (%) |
|---|---|---|
| Discrete | 38.71 | 87.95 |
| Continuous | 47.21 | 89.07 |

**Note.** MAPE calculated as average absolute percent change across years 1 to 10.

Overall, the two models are well-aligned within the first three years in terms of mean default probability, credit VaR estimates, and $L^1$ norm. The divergences in later years suggest that the continuous Markov process is more applicable for short-term credit risk assessment. This is further supported by the sensitivity analysis and research from (Kiefer and Larson, 2004). In general, the continuous model provides a close approximation to the discrete model over the short term while offering slightly more conservative credit risk estimates, making it a more robust model for credit risk analysis.

## Limitations

While Markov models provide a structured, analytic approach to assessing credit risk, there are inherent limitations arising from simplifying assumptions, data constraints, and economic sensitivity that restrict their applicability. These limitations affect both the continuous and discrete models, potentially leading to missed estimates of the true credit risk scenario under certain conditions.

The estimation of default probabilities from historical transition matrices is constrained by the number of issuer-years (the cumulative total of all years tracked in the data set), leading to limited detail in the transition probability estimates (RiskMetrics Group, 2007). This limits the precision of estimated default probabilities and leads to unrealistically long expected time to default. This can be seen when solving the discrete matrix $P$ for time to absorption. From the results in Table 20, it would take 137.28 years for a AAA bond to default. This suggests that probabilities listed as zero on the S&P Global transition matrix may not truly be zero, and reflect the limitations of using solely historical data.

**Table 20**

*Expected Time to Default by Credit Rating*

| Credit Rating | Expected Time to Default (Years) |
|---|---|
| AAA | 137.28 |
| AA | 129.33 |
| A | 119.42 |
| BBB | 101.38 |
| BB | 68.69 |
| B | 40.14 |
| CCC | 16.62 |

**Note.** Values represent the expected number of years to default, conditional on initial credit rating.

As observed from the sensitivity analysis, the model is highly sensitive to economic conditions. The large deviations in MAPE of credit VaR from recession (~88%) and expansion periods (~39%) relative to the baseline indicate that the model outputs may not be accurate without taking the business cycle into account. This is especially the case in downturn or recession scenarios, where there are much higher risks of default, and underestimating risk could lead to losses far exceeding expectations. This suggests that, while the model remains reliable as an average, it may fail to capture short-term (under 1 year) credit risk accurately without accounting for the underlying economic conditions.

Finally, a structural limitation arises for the use of the logarithmic approximation of the generator matrix for the continuous-time Markov process. This approximation assumes that at most one rating transition can occur per year (Jarrow et al., 1997), which does not truly align with real-world credit ratings. Other approximation methods exist without this simplifying assumption and may better approximate the generator matrix with added computational complexity, such as the algorithm presented by Israel et al. (2001), involving the eigenvalues of the matrix $P$. However, given the scope of this research, the logarithmic approximation method was chosen as an appropriate simplifying assumption.

**Approach**

To address the research question - comparing how Markov models perform when modeling the credit risk in a fixed bond portfolio - a discrete and continuous-time Markov model was implemented in MATLAB. Key risk metrics were chosen from Malhotra (2022): future rating transitions, cumulative default probabilities, and credit VaR.

For model validation and comparison over the 10-year horizon, the KL divergence and $L^1$ norms were calculated following the steps outlined by Israel et al. (2001). The $KL$ divergence was calculated as

$$D_{\mathrm{KL}}(P \parallel Q) = \sum_{x \in X} P(x) \log \left( \frac{P(x)}{Q(x)} \right) \tag{16}$$

where $P$ and $Q$ are row-vector probability distributions. Applying this to each row produces the results shown previously in Table 11. $L^1$ norm was calculated as the sum of the absolute values of the matrix $P^t - e^{Q \cdot t}$ where $P$ is the discrete model, $Q$ is the approximated generator, and $t$ is the time in years.

The non-absorbing steady states used in its benchmark against the debt market were calculated as

$$\pi P = \pi, \quad \text{with} \quad \sum_{i=1}^{n} \pi_i = 1 \tag{17}$$

where $\pi$ is the steady-state distribution and $P$ is the modeled transition matrix. According to RiskMetrics Group (2007), the non-absorbing steady states of a transition matrix (when calculating credit VaR) should broadly resemble the current debt market. Data for the debt market is derived from S&P Global (2024).

Cumulative default rates were calculated over 10 years, corresponding to the last column of the transition matrix at year $t$, excluding the default row. Future probabilities for the discrete and continuous models were calculated using (3) and (4), respectively. To summarize how each model estimated the risk of default in general, the mean cumulative default rate was taken for each year. To compare each model's estimates of default, the percent change relative to the discrete models was calculated.

Credit VaR was calculated following the CreditMetrics Monte Carlo simulation described by Han (2008). Annual stock return rates for each company in the bond portfolio over 10 years were obtained through Yahoo Finance and stored in an Excel spreadsheet. These returns were used with (11) and (12) to build the Z threshold matrix, correlated to each transition matrix at time $t$. Future values of each bond were calculated according to Hull (2015), specifically, (13), (14), and (15). Using 10,000 simulations for each year, portfolio values were sampled from the multivariate distribution with mean and covariance of stock returns as inputs. Bonds were revalued compared to the Z matrix to model transitions, and credit VaR was determined at a 95% confidence level using (6).

Finally, a sensitivity analysis was conducted to see how each model performed under different economic conditions. Recession and expansion matrices were obtained from S&P Global's credit reports for 2008 (recession) and 2021 (expansion). Both discrete and continuous models in each scenario were used to examine $L^1$ norm, cumulative default probabilities, and credit VaR. Each result was then compared against the baseline to get a measure of sensitivity.

**Applicability**

These Markov models directly address the research question by providing a structured and analytical comparison of how they estimate credit risk and default probabilities over time. Both the discrete-time Markov chain and continuous-time Markov process directly model the evolution in cumulative default probabilities and credit VaR. These outcomes provide interpretable metrics for evaluating and comparing their associated credit risks, with the continuous Markov process being a more conservative estimate. These models can be used by stakeholders in the corporate bond markets (banks, insurance companies, and asset managers), who rely on credit risk estimates for portfolio management.

This modeling approach is directly aligned with the research question, which aims to determine whether a continuous model can appropriately be used to model credit risk compared to a traditional discrete yearly transition matrix. As the results have shown, the models maintain similar behavior within three years, suggesting that the continuous Markov process can reliably be used as a model of credit risk within the bounds of the likelihood ratio test outlined by Kiefer and Larson (2004). Beyond the scope of this project, modeled transition matrices and the associated Monte Carlo simulation can be scaled to any size portfolio, providing real-world credit risk metrics. Furthermore, due to its continuous nature, the Markov process model can be used in risk scenarios under one year. Taking the current state of the business cycle into account can further increase the robustness of the continuous Markov process when modeling credit risk below a one-year horizon.

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